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SINGULAR PERTURBATIONS IN ONE-DIMENSIONAL UNSTEADY MOTION OF A REAL GAS

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Navier-Stokes equations for one-dimensional motion of gas are reduced to a special dimensionless form convenient for investigations involving a perturbation front. In new variables the transition from limit conditions of motion of an inviscid non-heat-conducting gas to the case of small but finite coefficients of viscosity and thermal conductivity, which is simulated by a perfect gas with singular perturbations induced by the indicated dissipative factors. We establish the inevitability of existence of two regions of singular perturbations, the neighborhood of the perturbation front and that of the point (line, surface) where the investigated motion is generated. The derivation of equations for both boundary layers, which is valid for a fairly general statement of problems of this kind, is presented and conditions of merging with the external (adiabatic) flow are formulated. Examples of computation of motion in boundary layers in problems of piston and point explosion are presented.

1. Transformation of Navier-Stokes equations. Let us consider the one-dimensional unstable motion of a real perfect gas with constant specific heats c_p and c_v and, also, the Prandtl number $\sigma = \mu c_p / \lambda$, where μ and λ are the coefficients of viscosity and thermal conductivity, respectively.

Using the dimensionless parameter ν which defines the kind of motion symmetry ($\nu = 1, 2, 3$), we write the Navier-Stokes equations in the form

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial v}{\partial r} + \frac{\nu-1}{r}v\right) = 0 \tag{1.1}$$

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}\right) + \frac{\partial p}{\partial r} = \frac{4}{3}\frac{\partial}{\partial r}\left[\mu\left(\frac{\partial v}{\partial r} - \frac{\nu-1}{2}\frac{v}{r}\right)\right] +$$

$$2(\nu-1)\frac{\mu}{r}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)$$

$$\rho\left(\frac{\partial e}{\partial t} + v\frac{\partial e}{\partial r}\right) + p\left(\frac{\partial v}{\partial r} + \frac{\nu-1}{r}v\right) = \frac{\varkappa}{\sigma}r^{1-\nu}\frac{\partial}{\partial r}\left(r^{\nu-1}\mu\frac{\partial e}{\partial r}\right) +$$

$$2\mu\left\{\left(\frac{\partial v}{\partial r}\right)^2 + (\nu-1)\frac{v^2}{r^2} - \frac{1}{3}\left[\frac{\partial v}{\partial r} + (\nu-1)\frac{v}{r}\right]^2\right\} \qquad \left(\varkappa = \frac{c_p}{c_v}\right)$$

$$p = (\varkappa - 1)\rho e, \quad \mu = Ae^n \, (A, \ n = \mathrm{const})$$

We assume that the density distribution of the gas in space conforms to the law

$$\rho_1 = Br^{-\omega} \quad (B, \ \omega = \mathrm{const}) \tag{1.2}$$

We furthermore assume that the unperturbed gas is at rest at zero values of temperature and pressure. This makes it possible to consider the region of perturbed motion to be of finite dimensions and bounded by surface $r = r_f(t)$ which is called the perturbation front. Function $r_f(t)$ is determined in the course of solution, which does not prevent us from using it for transforming Eqs. (1.1) to a new form that is more convenient for solving boundary value problems and for investigating the extreme modes of motion.

We introduce the notation $dr_f / dt = U$ and substitute the new independent dimensionless variables

$$\eta = \frac{r}{r_f}, \quad \chi = \frac{A}{(\varkappa - 1)^n} \frac{U^{2n-1}}{Br_f^{1-\omega}} \tag{1.3}$$

for r and t.

The choice of the second argument (1.3) is based on two considerations. First, it must depend only on time and be a continuous monotonic differentiable function of t (fulfilment of these conditions is proved later). Second, in specifying all physical parameters of gas, except μ and λ, we stipulate that argument χ must vanish simultaneously with these coefficients. These considerations and the condition for it to be dimensionless yield for χ the definition (1.3).

For the unknown functions in Eqs. (1.1) we also substitute new dimensionless functions defined by formulas

$$\begin{gathered} v = UV(\eta, \chi), \quad \rho = Br_f^{-\omega}R(\eta, \chi), \; p = Br_f^{-\omega}U^2P(\eta, \chi), \\ e = (\varkappa - 1)^{-1}U^2 N(\eta, \chi), \quad \mu = \chi Br_f^{-\omega}UN^n(\eta, \chi) \end{gathered} \tag{1.4}$$

To transform Eqs. (1.1) we need the quantity (subscript t denotes differentiation with respect to time)

$$Z = U_t U^{-2} r_f \tag{1.5}$$

which is generally not known a priori, depends on χ and is constant for any χ. It is important to take into account that for $\chi = 0$, i.e. in the absence of viscosity, $Z(0) = Z_0$ is known and conforms to the law of motion of the front in the extreme mode, when the latter is transformed into a shock wave.

Reverting to formulas (1.3), we note that in conformity with the definition of the new argument χ we have

$$d \ln \chi / dt = Ur_f^{-1}K(\chi), \quad K(\chi) = \omega - 1 + (2n - 1) Z \tag{1.6}$$

Using the notation of (1.5) and (1.6) and introducing new variables defined in (1.3) and (1.4) into Eqs. (1.1), we reduce the latter to the form

$$\begin{aligned} &(V - \eta)\frac{\partial R}{\partial \eta} - \omega R + K\chi \frac{\partial R}{\partial \chi} + R\frac{\partial V}{\partial \eta} + \frac{\nu - 1}{\eta} RV = 0 \\ &R\left[ZV + (V - \eta)\frac{\partial V}{\partial \eta} + K\chi\frac{\partial V}{\partial \chi}\right] + \frac{\partial P}{\partial \eta} = \\ &\quad \frac{4}{3}\chi\frac{\partial}{\partial \eta}\left[N^n\left(\frac{\partial V}{\partial \eta} - \frac{\nu - 1}{2}\frac{V}{\eta}\right)\right] + 2(\nu - 1)\chi\frac{N^n}{\eta}\left(\frac{\partial V}{\partial \eta} - \frac{V}{\eta}\right) \\ &R\left[2ZN + (V - \eta)\frac{\partial N}{\partial \eta} + K\chi\frac{\partial N}{\partial \chi} + (\varkappa - 1)N\left(\frac{\partial V}{\partial \eta} + \frac{\nu - 1}{\eta}V\right)\right] = \\ &\quad \frac{\varkappa}{\sigma}\chi\eta^{1-\nu}\frac{\partial}{\partial \eta}\left(\eta^{\nu-1}N^n\frac{\partial N}{\partial \eta}\right) + 2(\varkappa - 1)\chi N^n\left\{\left(\frac{\partial V}{\partial \eta}\right)^2 + \right. \\ &\quad \left.(\nu - 1)\frac{V^2}{\eta^2} - \frac{1}{3}\left[\frac{\partial V}{\partial \eta} + (\nu - 1)\frac{V}{\eta}\right]^2\right\}, \quad P = RN \end{aligned} \tag{1.7}$$

Equations (1.7) represent the dimensionless equivalent of the Navier-Stokes equations, and can be used for investigating and deriving the solution of a fairly wide class of problems of one-dimensional unstable motion of real gas in the presence of a perturbation front. At the limit $\chi = 0$ Eqs. (1.7) are transformed into a system that defines the self-similar adiabatic motion of perfect gas in the presence of shock waves (see [1]). However the transition from $\chi = 0$ to small but finite χ is associated with the appearance of regions of singular perturbation that are similar to the boundary layer on bodies in an external stream. Besides the purely formal indication (small coefficient at the leading derivative) the singular character of behavior of solutions of Eqs. (1.7) in the neighborhood of certain surfaces $\eta = \text{const}$ and small χ is explained by that all solutions of the limit self-similar problem cannot simultaneously satisfy all boundary conditions of the complete problem.

It can be readily shown that in the case of the considered class of one-dimensional motions at least two regions of singular perturbation of the boundary layer kind must develop. One of these lies in the neighborhood of the perturbation front, where the transition from hydrodynamic parameters related to a strong shock wave to parameters of the unperturbed gas must occur for small χ. The second boundary layer lies at the point (line, surface) where the investigated motion itself is generated. Solutions of the dimensionless variant of Euler's equations satisfy at that point only the condition for velocity, while the specification of temperature or of the heat flux results in the appearance of related transition region.

A qualitative and quantitative analysis of the two indicated boundary layers is presented below.

2. The boundary layer of the perturbation front. Let us refine the boundary conditions that must be satisfied at the perturbation front. That front is a surface of weak discontinuity which does not contain sources of mass, momentum, or energy and in which basic variables are the same as in an unperturbed region. With the use of notation of Eqs. (1.7) we obtain boundary conditions of the form

$$R(1, \chi) = 1, \quad P(1, \chi) = N(1, \chi) = V(1, \chi) = 0 \tag{2.1}$$
$$\chi (N^n \partial N / \partial \eta)_{\eta=1} = \chi (N^n \partial V / \partial \eta)_{\eta=1} = 0$$

Conditions (2.1) must be supplemented by the conditions of boundary layer merging with the external (inviscid) stream. This will be done later, after the introduction of deformed variables of the boundary layer.

Assuming that surface $\eta = 1$ is the inner boundary of the front boundary layer, we change argument η so that the viscous and basic convection terms of Eqs. (1.7) become of the same order with respect to χ. This is feasible, if we assume

$$\eta = 1 - \chi \eta_* \tag{2.2}$$

The scale of unknown variables remains unchanged, but we add to the notation subscript * . After this, with the use of substitution (2.2) followed by passing to limit $\chi \to 0$, from Eqs. (1.7) we obtain

$$(R_* V_*)' - R_*' = 0, \quad R_* (V_*' - V_* V_*') - P_*' = {}^4/_3 (N_*^n V_*')' \tag{2.3}$$
$$R_* (N_*' - V_* N_*') - (\varkappa - 1) P_* V_*' = (\varkappa / \sigma) (N_*^n N_*')' +$$
$${}^4/_3 (\varkappa - 1) N_*^n (V_*')^2, \quad P_* = R_* N_*$$

where the prime denotes a derivative with respect to η_*. The group of "inner" boundary conditions is readily derived from (2.1), and with their use the order of system (2.3) can be easily reduced, yielding

$$R_* = 1 / (1 - V_*), \quad {}^4/_3 N_*{}^n V_*' = V_* - R_* N_* \qquad (2.4)$$
$$(\varkappa / \sigma) N_*{}^n N_*' = N_* - {}^1/_2 (\varkappa - 1) V_*{}^2$$

System (2.4) corresponds to the system of the first approximation equations that define the shock wave structure in a hypersonic flow obtained in [2]. The properties of solutions of this system are analogous to those for the structure of a stationary shock wave of arbitrary intensity which was thoroughly investigated by many authors [3—8]. Note that for $\sigma = {}^3/_4$ system (2.4) admits (for certain n) an analytic solution or can be reduced to a numerical quadrature.

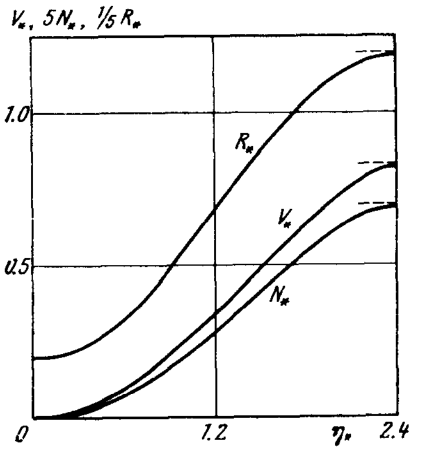


Fig. 1

The conditions of merging with the external adiabatic stream are the "external" boundary conditions for Eqs. (2.4). In the considered case values of all hydrodynamic parameters of the boundary layer must for $\eta_* \to \infty$ be equal to those of the adiabatic stream at the shock wave, i.e. for $\eta_* \to \infty$ we have

$$R_* \to \frac{\varkappa + 1}{\varkappa - 1}, \qquad (2.5)$$

$$V_* \to \frac{2}{\varkappa + 1}, \quad N_* \to \frac{2(\varkappa - 1)}{(\varkappa + 1)^2}$$

accurate to within exponentially attenuating terms.

Certain peculiarities of Eqs. (2.4) and their boundary conditions should be noted. First, it will be seen that the motion of gas in the front boundary layer is independent of both the problem dimensionality and of parameter density variation ω. Second, for $\eta_* = 0$ the solutions of system (2.4) have a singularity, and the numerical solution of that system necessitates the use of asymptotic expansions (see [2]). For $n = {}^1/_2$ and $\varkappa = 2\sigma$ the first terms of these expansions are of the form

$$N_* = {}^1/_{16}\eta_*{}^2 + O(\eta_*{}^4), \quad V_* = {}^3/_{16}\eta_*{}^2 + C\eta_*{}^3 + O(\eta_*{}^4) \qquad (2.6)$$
$$R_* = 1 + {}^3/_{16}\eta_*{}^2 + C\eta_*{}^3 + O(\eta_*{}^4)$$

Formulas (2.6) ensure the fulfilment of all boundary conditions for $\eta_* = 0$ and an approximate fulfilment of Eqs. (2.4) for small η_*, while by a suitable selection of the constant C we can satisfy the asymptotic conditions (2.5). Profiles of velocity, temperature and density variation in the front boundary layer, obtained by numerical methods for $n = {}^1/_2$, $\varkappa = 1.4$ and $\sigma = 0.7$ are shown in Fig. 1.

3. Asymptotic formulas for the external stream valid in the perturbation source neighborhood. To determine the pattern of motion in the boundary layer in the neighborhood of the perturbation source, we need asymptotic formulas for the parameters of the external adiabatic stream in that region. Many formulas of this kind have been published (see [1, 9, 10], however, owing to the absence of a complete set of these for various perturbation sources and various values of ν and ω, we shall present the derivation of such formulas in a most general form.

We assume that the adiabatic self-similar motion is generated at point $\eta = b$ ($b=0$ corresponds to the particular case of point explosion). The input equations are derived from (1.7) by setting in these $\chi \equiv 0$; they are of the form

$$V' = \frac{Z_0\eta(\eta - V)V + (2Z_0 - \omega)\eta N + \varkappa(\nu - 1)NV}{\eta[(\eta - V)^2 - \varkappa N]} \tag{3.1}$$

$$\frac{R'}{R} = \frac{V' + (\nu - 1)V/\eta - \omega}{\eta - V}, \qquad \frac{N'}{N} = \frac{2Z_0 + (\varkappa - 1)[V' + (\nu - 1)V/\eta]}{\eta - V}$$

where the prime denotes differentiation with respect to η.

We introduce the notation $\lambda = \eta - b$ and assume that for small λ we have in all cases

$$V = b + c_V\lambda \tag{3.2}$$

where constant b is specified, and for the determination of coefficients c_V from (3.1) we have the formula

$$c_V = (2Z_0 - \omega)/\varkappa - (\nu - 1)\Pi \tag{3.3}$$

$$\Pi = \lim_{\lambda\to 0}\frac{V}{\eta} = \lim_{\lambda\to 0}\frac{b + c_V\lambda}{b + \lambda} = \begin{cases} 1, & b \neq 0 \\ c_V, & b = 0 \end{cases}$$

Having determined c_V we can substitute $V' = c_V$ into the second and third equations of system (3.1); using (3.3) we obtain

$$R = c_R(\eta - b)^m, \qquad N = c_N(\eta - b)^{-m} \tag{3.4}$$

$$m = -\frac{2Z_0 + \omega(\varkappa - 1)}{\varkappa(1 - c_V)}$$

It is usually not possible to determine coefficients c_R and c_N by analyzing only the limit form of Eqs. (3.1), however their explicit expressions are not essential for further investigations.

4. Boundary layer in the perturbation source neighborhood. We assume that at the point which represents the source of perturbations the analog of velocity is $V = V_s(\chi)$ with $V_s(0) = b$ (see Sect. 3). As the "thermal" boundary condition at that point we take the condition for zero heat flux there, i.e.

$$\chi\,(N^n\partial N/\partial\eta)_{\eta=\eta_s} = 0 \tag{4.1}$$

We pass in Eqs. (1.7) to variables

$$V = V_s + V^\circ\chi^\beta, \quad \eta = b + \eta^\circ\chi^\beta, \quad N = N^\circ\chi^{-\alpha}, \quad R = R^\circ\chi^\alpha \tag{4.2}$$

and then, as in Sect. 2, pass to the limit $\chi \to 0$. One of the conditions for the determination of constants α and β reduces to the requirement that after transformation (4.2) of the equation of energy its terms which correspond to convective heat transfer and thermal conductivity be of the same order with respect to χ. This reduces to

$$1 - (n + 1)\alpha = 2\beta \tag{4.3}$$

The second condition consists in that for $\eta^\circ \to \infty$ solutions of transformed equations must convert into the asymptotic formulas (3.4) independently of the fixed value of χ, in other words, the merging of solution for the boundary layer with the external stream must be ensured (see [11]). Using (3. 4) and (4. 2) we can write

$$N^{\circ}_{\eta^{\circ}\to\infty} = N_{\eta\to b}\chi^{\alpha} = c_N(\eta - b)^{-m}\chi^{\alpha} = c_N\eta^{\circ -m}\chi^{\alpha - m\beta} \tag{4.4}$$

and evidently assume

$$\alpha = m\beta \tag{4.5}$$

From (4. 3) and (4. 5) we obtain

$$\alpha = \frac{m}{2 + m(n+1)}, \quad \beta = \frac{1}{2 + m(n+1)} \tag{4.6}$$

where m is determined by formula (3. 4).

To pass to limit after the substitution of (4. 2) into (1. 7) it is necessary to know the behavior of the relationship

$$\lim_{\chi\to 0}\frac{V - \eta}{\eta - b} = \lim_{\chi\to 0}\frac{V_s - b + (V^{\circ} - \eta^{\circ})\chi^{\beta}}{\eta^{\circ}\chi^{\beta}} = \frac{V^{\circ} - \eta^{\circ}}{\eta^{\circ}} \tag{4.7}$$

In the computation of the limit it is assumed here that $V_s - b$ decreases with decreasing χ more rapidly than χ^{β}. The validity of this assumption must be tested after refinement of the form of function $V_s(\chi)$.

It should be noted that at the limit $\chi \to 0$ the behavior of some of the functions differs for $b = 0$ and $b \neq 0$. To maintain the unity of notation it is expedient to introduce symbols of step functions as defined in

$$\varphi(b) = \begin{cases} 0, & b = 0, \\ 1, & b \neq 0, \end{cases} \quad \psi(b) = 1 - \varphi(b) \tag{4.8}$$

Taking into account (4. 7) and notation (4. 8), after the substitution of variables in Eqs. (1. 7) and passing to limit $\chi \to 0$ we obtain

$$\begin{gathered}(V^{\circ} - \eta^{\circ})R^{\circ\prime} + [\alpha(\omega - 1) + \alpha(2n-1)Z_0 - \omega + (\nu - 1)\times \\ \varphi(b)]R^{\circ} + (\nu - 1)\psi(b)R^{\circ}V^{\circ}/\eta^{\circ} + R^{\circ}V^{\circ\prime} = 0 \\ [2Z_0 - \alpha(\omega - 1) - \alpha(2n-1)Z_0 + (\varkappa - 1)(\nu - 1)\varphi(b)]R^{\circ}N^{\circ} + \\ (V^{\circ} - \eta^{\circ})R^{\circ}N^{\circ\prime} + (\varkappa - 1)[\psi(b)(\nu - 1)V^{\circ}/\eta^{\circ} + V^{\circ\prime}]R^{\circ}N^{\circ} = \\ (\varkappa/\sigma)[(N^{\circ n}N^{\circ\prime})' + (\nu - 1)\psi(b)N^{\circ n}N^{\circ\prime}/\eta^{\circ}], \quad (R^{\circ}N^{\circ})' = 0\end{gathered} \tag{4.9}$$

where the prime denotes differentiation with respect to η°.

Equations (4. 9) were derived on the important assumption that $\alpha + 2\beta > 0$, which is justified in all practically interesting cases.

System (4. 9) admits the energy integral for both $b \neq 0$ and $b = 0$. For $b \neq 0$ that integral is obtained directly by replacing the derivative $N^{0\prime}$ in the left-hand part of the system's second equation with the use of third and first equations. For $b = 0$ (point explosion) $Z_0 = -\nu/2$, which corresponds to this case, is to be used. Recalling formula (3. 3) and using boundary conditions (4. 1) together with condition $V^{\circ}(0) = 0$, we can represent the energy integral in the general-purpose form

$$N^{\circ n}N^{\circ\prime} = \sigma R^{\circ}N^{\circ}(V^{0} - c_V\eta^{\circ}) \tag{4.10}$$

Taking into account that by virtue of the last of Eqs. (4. 9) $R^{\circ}N^{\circ} = c_P$ (the constant known from self-similar solution) and introducing the new unknown function

$$\Phi = N^{\circ n+1} \tag{4.11}$$

it is possible to transform Eqs. (4.9) to a system of two first order equations

$$\Phi' = (n + 1)\, c_P \sigma\, (V^\circ - c_V \eta^\circ) \tag{4.12}$$

$$V^{\circ\prime} = -(\nu - 1)\, \psi(b)\, V^\circ / \eta^\circ + \sigma c_P (V^\circ - \eta^\circ)(V^\circ - c_V\eta^\circ) / \Phi -$$
$$\alpha\, [\omega - 1 + (2n - 1) Z_0] + \omega - (\nu - 1)\, \varphi(b)$$

System (4.12) is to be integrated with conditions

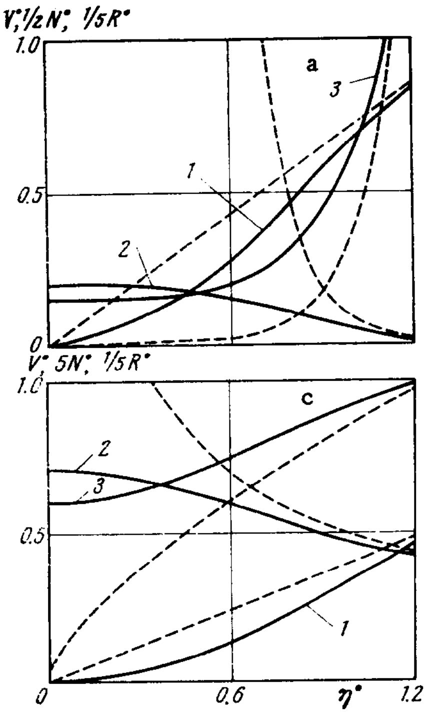
$$V^\circ(0) = 0, \quad V^\circ(\infty) = c_V \eta^\circ, \quad \Phi(\infty) = c_N^{n+1} \eta^{\circ\, \sigma - m(n+1)} \tag{4.13}$$

which does not present any fundamental difficulties. Having obtained a numerical solution for functions $V^\circ(\eta^\circ)$ and $\Phi(\eta^\circ)$ we can obtain

$$N^\circ(\eta^\circ) = \Phi(\eta)^{1/(n+1)}, \quad R^\circ(\eta^\circ) = c_P / N^\circ(\eta^\circ) \tag{4.14}$$

The last two conditions in (4.13) are basically the asymptotic equations for merging with the solution for the external stream, since it is these conditions that were used in determining formula (4.5).

Let us indicate some qualitative singularities of the boundary layer at the perturbation source. First, we note that similarly to the aerodynamic boundary layer the pressure across it is constant (the front boundary layer does not have this property). Second, the thickness of the considered boundary layer for fixed χ is of different order than the boundary layer at the front. To determine the order of thickness of each layer it is sufficient to compare formulas (2.2) and (4.2) for the transformation of the independent variable. These formulas show that the front layer thickness is of order $\delta^\circ = O(\chi)$, while at the perturbation source it is $\delta^\circ = O(\chi^\beta)$. In the majority of cases β is smaller than unity, and can be sometimes of order 0.01, which means that the boundary layer at the perturbation source



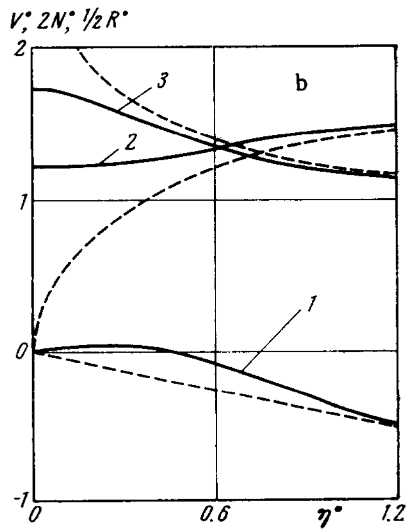


Fig. 2

is considerably thicker than the similar layer at the front. The former is of noticeable thickness even for very small χ, while the thickness of the front layer can be considered to be negligibly small.

Note that in the particular case of the problem of point explosion Sychev [12] had investigated the boundary layer at the perturbation source for $n = 1$, $\nu = 3$, $\omega = 0$ and $\varkappa = {}^7/_5$.

Results of numerical calculations with the use of Eqs. (4.12)—(4.14) are shown in Fig. 2, where the variable η° appears on the axis of abscissas and V°, N° and R^c are plotted along the axis of ordinates with the curves of these denoted by numerals *1*, *2* and *3* , respectively. The dash lines represent asymptotic curves of the external stream. Curves for the case of point explosion are shown in Fig. 2, a for $n = {}^1/_2$, $\nu = 3$, $\omega = 0$, $\varkappa = {}^7/_5$, $Z_0 = -{}^3/_2$ and $\sigma = 0.7$, with $m = 7.5$, $c_N = 0.1634$, $c_V = {}^5/_7$, $c_P = 0.3046$, $\alpha = 0.5660$ and $\beta = 0.0755$. Figure 2, b relates to the problem of a piston moving in accordance with the law $r_p = Ct^{10/11}$, for $n = {}^1/_2$, $\nu = 3$, $\omega = 2$, $\varkappa = {}^7/_5$, $Z_0 = -0.1$ and $\sigma = 0.7$, with $m = -0.3$, $c_N = 0.3513$, $c_V = -{}^3/_7$, $c_P = 0.8495$, $\alpha = -0.1935$ and $\beta = 0.6452$. Figure 2, c relates to the case of a piston moving according to the law $r_p = Ct^{3/4}$, $n = 1$, $\nu = 1$, $\omega = 0$, $\varkappa = {}^5/_3$, $Z_0 = -{}^1/_8$ and $\sigma = {}^2/_3$, with $m = {}^2/_3$, $c_N = 0.0988$, $c_V = 0.4$, $c_P = 0.4227$, $\alpha = 0.2$ and $\beta = 0.3$.

These examples were chosen so as to show the possible variety of conditions in the external stream and the related versions of boundary layers. These calculations and numerical data confirm the above qualitative conclusions.

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ON INTEGRALS OF EQUATIONS OF UNSTABLE NEARLY SELF-SIMILAR FLOWS

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One-dimensional or nearly one-dimensional unstable motions of perfect gas are considered. Integrals admitted by the system of equations defining such motions are examined. Since the existence of integrals is associated with some law of conservation, i. e. with some divergent form of presentation of equations of the input system, it is possible by examining all divergent equations of gas-dynamics to derive certain new integrals not previously considered.

1. As the basic system we select the continuity equation, the Euler equation, and the equation of energy conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_k}{\partial x_k} = 0 \tag{1.1}$$

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial}{\partial x_k}(\rho v_i v_k + \delta_{ik} p) = 0 \tag{1.2}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v_i^2 + \frac{p}{\varkappa - 1}\right) + \frac{\partial}{\partial x_k}\left(\frac{1}{2}\rho v_k v_i^2 + \frac{\varkappa}{\varkappa - 1} v_k p\right) = 0 \tag{1.3}$$

where subscripts i and k assume the values 1, 2, 3, and recurrent subscripts indicate summation.

Below we refer to certain equations as being of divergent form, if their variables appear as derivatives, e. g. Eqs. (1.1)–(1.3). Equations of divergent form are also called laws of conservation.

Instead of Eq. (1.3) it is possible to use the equation of conservation of entropy of a particle

$$\frac{\partial S}{\partial t} + v_k \frac{\partial S}{\partial x_k} = 0, \quad S = \frac{p}{\rho^\varkappa} \tag{1.4}$$

We denote by $A(S)$ an arbitrary function of S and by $A'(S)$ its derivative with respect to S. We multiply Eqs. (1.1) and (1.4) by $A(S)$ and $\rho A\ (S)$, respectively, and add the results. We obtain

$$\frac{\partial \rho A(S)}{\partial t} + \frac{\partial}{\partial x_k}(\rho v_k A(S)) = 0 \tag{1.5}$$

Equation (1.5) is of divergent form and contains an arbitrary function of entropy.

Let us transform Eqs. (1.1) and (1.2). For convenience we introduce the following notation: